Universal AI: AIXI

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Iliad Intensive

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- **Universal** choice: $\mathcal{M} =$ all computable environments, prior $w_\nu = 2^{-K(\nu)}$ (K is the "complexity" of the environment). Assumption "$\mu \in \mathcal{M}$" becomes "*the universe is computable*".

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Class $\mathcal{M} = \{\nu_1, \nu_2, \dots\}$ with prior weights $w_\nu > 0$, $\sum_\nu w_\nu = 1$.

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- **Key fact (mixture is a predictor):** its one-step prediction is a *posterior-weighted* average of the members' predictions (you prove this today!)

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- **Posterior update is multiplicative:** $w(\nu \mid x_{1:t}) = w(\nu \mid x_{<t}) \cdot \dfrac{\nu(x_t \mid x_{<t})}{\xi(x_t \mid x_{<t})}$: we reweight by how well ν predicted vs. ξ (Bayes' rule!)

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- Spaces: actions $\mathcal{A}$, observations $\mathcal{O}$, rewards $\mathcal{R} \subset [0,1]$, percepts $\mathcal{E} = \mathcal{O} \times \mathcal{R}$, and (finite) histories $\mathcal{H}^* := (\mathcal{A} \times \mathcal{E})^* := \cup_{t=0}^{\infty} (\mathcal{A} \times \mathcal{E})^t$.

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- **No Markov / MDP assumption!** Both the policy and environment can depend on all prior percepts.

The three players

Policy π

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Interaction measure ν^π

Run π inside ν and multiply along the history:

$$\nu^\pi(æ_{t:m} \mid æ_{<t}) := \prod_{k=t}^{m} \pi(a_k \mid æ_{<k})\, \nu(e_k \mid æ_{<k} a_k).$$

Value, optimality, and AIXI

Value function (assuming geometric discount γ)

$$\begin{aligned}
V_\nu^{\pi,m}(æ_{<t}) &= (1-\gamma)\,\mathbb{E}_\nu^\pi[G_{t-1:m} \mid æ_{<t}] \text{ with } G_{t-1:m} = \sum_{k=t}^{m} \gamma^{k-t}\, r_k \\
&= (1-\gamma)\sum_{æ_{t:m}} \nu^\pi(æ_{t:m} \mid æ_{<t}) \sum_{k=t}^{m} \gamma^{k-t}\, r_k
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- **Optimal value** $V^*_\nu := \sup_\pi V^\pi_\nu$; an optimal policy π^*_ν satisfies $V^{\pi^*_\nu}_\nu = V^*_\nu$.
- **Bayes-optimal policy** $\pi^*_\xi \in \arg\max_\pi V^\pi_\xi$: acts optimally w.r.t. the *mixture* ξ.

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Intuition: *The Bayes-optimal agent with prior according to Occam's razor: Act optimally according to environments that you believe to be in, assuming the simplest environments consistent with observations are more likely to be true.*

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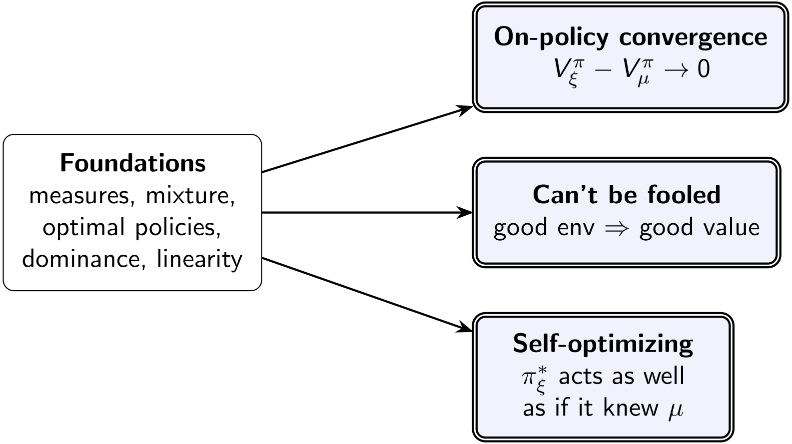
AIXI, explicitly

max over actions, $\sum$ over percepts, choosing actions that maximise the ξ-weighted discounted return:

$$a_t = \arg\max_{a_t} \lim_{m\to\infty} \sum_{e_t} \max_{a_{t+1}} \sum_{e_{t+1}} \cdots \max_{a_m} \sum_{e_m} \xi(e_{t:m} \mid æ_{<t}\, a_{t:m}) \sum_{k=t}^{m} \gamma^{k-t} r_k$$

The three main AIXI results



- All three: the Bayesian agent inherits the guarantees it would have if it knew the truth.

Result 1: On-policy value convergence

Theorem

For any $\mu \in \mathcal{M}$ and any policy π,

$$V_\xi^\pi(æ_{<t}) - V_\mu^\pi(æ_{<t}) \to 0 \qquad \mu^\pi\text{-almost surely.}$$

Intuition: *A policy that acts, pretending the environment is ξ, will eventually act as well as if it were in the true environment μ.*

Result 2: AIXI can't be fooled

Theorem (deterministic μ)

If the truth is reachable-good, $V^*_\mu(æ_{<t}) > \varepsilon$ along the played history, then

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Intuition: *Whenever good behaviour in μ is achievable, the Bayes-optimal agent secures at least a fraction proportional to the prior weight of μ.*

Result 3: Self-optimizing property (stretch goal)

Theorem

If *some* policy can learn to act optimally in every $\nu \in \mathcal{M}$, then π^*_ξ does too:

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Intuition: *The Bayes-optimal agent that is unaware of which environment it is in learns to act just as well as the best possible policy that already knows which environment it is in (μ).*

Takeaways

- **Solomonoff** solves induction: mix over all computable hypotheses, weighted by simplicity. Total prediction error $\leq K(\mu)\ln 2$.

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- **Why care for safety:** a precise, assumption-light model of idealized intelligence providing a lens on what optimal agents do. For example, we can show under what circumstances AIXI would wirehead [RO11], safely rather than recklessly explore [CCH20], or self-modify [OR11] as a model of what superintelligent agents would do.

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